

Math Reviewer for Civil Service Exam

1. Oil is pumped into a non-empty tank at a changing rate. The volume of oil in the tank doubles every minute and the tank is filled in 10 minutes. How many minutes did it take for the tank to be half full?

- A. 2 B. 5 C. 7 D. 8 E. 9

Answer: (e)

Solution: At the 10 minute mark, the tank has twice as much oil in it as it did at the 9 minute mark. So the tank became half-full at the 9 minute mark.

Area: Algebra

2. Alice and Bill are walking in opposite directions along the same route between A and B. Alice is going from A to B, and Bill from B to A. They start at the same time. They pass each other 3 hours later. Alice arrives at B 2.5 hours before Bill arrives at A. How many hours does it take for Bill to go from B to A?

- A. 6 B. 6.5 C. 7 d. 7.5 E. 8.5

Answer: (d)

Area: Algebra

Solution: Suppose that it takes Alice x hours to go from A to B. Then, it takes Bill $x + 2.5$ hours to go from B to A. Assuming that Alice and Bill maintain constant speed, we get that Alice's speed is $d=x$ and Bill's speed is $d=(x + 2.5)$, where d is the distance from A to B. Since Alice and Bill met 3 hours after they started walking,

$$3 \left(\frac{d}{x} + \frac{d}{x + 2.5} \right) = d.$$

Dividing by d and solving for x we get $x = 5$, so the answer is $5 + 2.5 = 7.5$ hours.

Area: Algebra

3. Two armies are advancing towards each other, each one at 1 mph. A messenger leaves the first army when the two armies are 10 miles apart and runs towards the second at 9 mph. Upon reaching the second army, he immediately turns around and runs towards the first army at 9 mph. How many miles apart are the two armies when the messenger gets back to the first army?

- A. 5.6 b. 5.8 C. 6 D. 6.2 E. 6.4

Answer: (e)

Solution: The messenger reaches the second army in one hour (the messenger and the second army advance at each other at a combined speed of 10 mph). At that time, the two armies are 8 miles apart. It takes 0.8 hours for the messenger to get back to the first army. At that time the armies are $8 - 0.8 - 0.8 = 6.4$ miles apart.

Area: Algebra

4. You play on a game show where a prize has been randomly put into one of five boxes, labeled A, B, C, D, and E, with each box equally likely to contain the prize. The boxes are closed when you first see them, and you guess that box A contains the prize. Two of the empty boxes from among the four other boxes are then opened. If these two boxes are C and E, what is the probability that box D contains the prize?

- A. $1/5$ B. $1/3$ C. $2/5$ D. $3/5$ E. $2/3$

Answer: (c)

Solution: The probability of the prize being in box A is equal to that of it being in any other box; hence this probability is $1/5$. The remaining $4/5$ is split between Box D and Box B; hence the probability that Box D contains the prize is $2/5$.

Area: Statistics

5. Six mountain climbers divide themselves into three teams for the final assault on a peak. One team has 3 climbers; the others have 1 and 2 climbers. All manners of different team deployments are considered, which team goes first, second, and third, and two deployments are considered different if the composition of any of the teams is different. (Disregard deployments within each team.) What is the total number of possible team deployments?

- A. 60 B. 180 C. 360 D. 720 E. none of the above

Answer: (c)

Solution: There are six ways to choose which team goes first, which goes second and which 6 goes third. Also, there are $\binom{6}{3} = 20$ ways to select three climbers out of the six for the three man team, and 3 ways to pick a climber out of the remaining three for the one man team. So, the total number of ways is $6 \cdot 20 \cdot 3 = 360$ ways.

Area: Statistics

6. How many positive integers less than or equal to 2013 are divisible by at least one of 3, 11, and 61?

- A. 3 B. 813 C. 1006 D. 1198 E. 2013

Answer: (b)

Solution: There are $2013/3 = 671$ multiples of 3 from 1 to 2013, $2013/11 = 183$ multiples of 11, and $2013/61 = 33$ multiples of 61. To avoid double counting we subtract the 61 multiples of 3 .11, the 11 multiples of 3 .61, and the 3 multiples of 61 .11. We have to add 1 for 2013 which was subtracted one too many times. Thus, the answer is $671 + 183 + 33 - 61 - 11 - 3 + 1 = 813$.

Area: Algebra

7. How many two digit prime numbers are there in which both digits are prime numbers? (For example, 23 is one of these numbers but 31 is not, since 1 is not a prime number.)

- A. 3 B. 4 C. 5 D. 8 E. 15

Answer: (b)

Solution: The second digit can only be 3 or 7, so the choice quickly narrows down to 23, 27, 33, 37, 53, 57, 73, and 77. Of these, 27, 33, and 57 are divisible by 3, and 77 by 7, leaving 23, 37, 53, and 73. It is easy to see that none of these is divisible by 2, 3, 5, or 7, and there is no need to look at greater prime divisors since $\sqrt{77} < 11$.

8. You own thirteen pairs of socks, all different, and all of the socks are individually jumbled in a drawer. One morning you rummage through the drawer and continue to pull out socks until you have a matching pair. How many socks must you pull out to guarantee having a matching pair?

A. 3 B. 12 C. 13 D. 14 E. 25

Answer: (d)

Solution: You might be unlucky and have the first thirteen socks all different, but then the 14th has to match one of them.

9. A jeweler has a 20 gram ring that is 60% gold and 40% silver. He wants to melt it down and add enough gold to make it 80% gold. How many grams of gold should be added?

A. 4 grams B. 8 grams C. 12 grams D. 16 grams E. 20 grams

Answer: (e)

Solution:

$$\begin{aligned} \text{Gold concentration} &= \text{weight of gold} / \text{total weight} \\ 0.80 &= \frac{20 \cdot 0.60 + x}{20 + x} \\ 12 + x &= 16 + 0.8x \\ 0.2x &= 4 \\ x &= 20 \text{ grams} \end{aligned}$$

10. Consider the following game. A referee has cards labeled A, B, C, and D, and places them face down in some order. You point to each card in turn, and guess what letter is written on the bottom. You guess each of A, B, C, and D exactly once (otherwise there is no chance of getting them all right!). You play this game once, and then the referee tells you that you guessed exactly n of the letters correctly. Which value of n is not a possible value of n?

A. 0 B. 1 C. 2 D. 3 E. 4

Answer: (d)

Solution: If the first three are correct, then by process of elimination the fourth has to be correct also. The same reasoning holds no matter when the three correct answers occur.

11. Consider the following game. A referee has cards labeled A, B, C, and D, and places them face down in some order. You point to each card in turn, and guess what letter is written on the bottom. You guess each of A, B, C, and D exactly once (otherwise there is no chance of getting them all right!). You play this game once, and then the referee tells you that you guessed exactly n of the letters correctly. Which value of n is not a possible value of n?

A. 0 B. 1 C. 2 D. 3 E. 4

Answer: (d)

Solution: If the first three are correct, then by process of elimination the fourth has to be correct also. The same reasoning holds no matter when the three correct answers occur.

12. On a test the passing students had an average of 83, while the failing students had an average of 55. If the overall class average was 76, what percent of the class passed?

- A. 44% B. 66% C. 68% D. 72% E. 75%

Answer: (e)

Solution: Let p = proportion that passed. Then $83p + 55(1 - p) = 76$, so $p = 21/28 = .75$

13. Jack and Lee walk around a circular track. It takes Jack and Lee respectively 6 and 10 minutes to finish each lap. They start at the same time, at the same point on the track, and walk in the same direction around the track. After how many minutes will they be at the same spot again (not necessarily at the starting point) for the first time after they start walking?

- A. 15 B. 16 C. 30 D. 32 E. 60

Answer: (a)

Solution 1: Experimenting with the numbers in turn, dividing 6 into 15 and 10 into 15, we get answers one whole number apart, so they are together again at 15 minutes.

14. A class has three girls and three boys. These students line up at random, one after another. What is the probability that no boy is right next to another boy, and no girl is right next to another girl?

- A. $1/20$ B. $1/12$ C. $1/10$ D. $3/10$ E. $1/2$

Answer: (c)

Solution: We pick the students in order. The first choice is always okay. There is a probability $3/5$ that the second choice is okay; for example, if you picked a boy first then there are two boys and three girls left. Similarly the probabilities that the remaining choices are okay is $1/2$, $2/3$, $1/2$, 1. The answer is $1/10$ the product of all these probabilities.

15. At a party, each person shakes hands with 5 other people. There are a total of 60 handshakes. How many people are at the party?

- A. 6 B. 12 C. 15 D. 24 E. 30

Answer: (d)

Solution: Let x be the total number of people. Then the total number of handshakes is $5x/2$ where we have to divide by 2 since every handshake is counted twice. Thus $5x/2 = 60$, which leads to $x = 24$.

16. Each of 10 students has a ticket to one of ten chairs in a row at a theater. How many ways are there to seat the students so that each student sits either in the chair specified on his/her ticket or in one right next to it? Each chair is to be occupied by exactly one student.

- A. 89 B. 144 C. 243 D. 512 E. 1024

Answer: (a)

Solution: Let $F(n)$ be the number of ways to seat n students in a row of n chairs as described in the problem. The student who has the ticket to seat number n has two options. He/she can either sit in his/her own chair, in which case the remaining $n - 1$ students can arrange themselves in $F(n - 1)$ ways. Alternatively, if this student takes the $(n - 1)$ st seat, the student with

the ticket to that chair is forced to take the n th seat, and the other $n - 2$ students can be seated in $F(n - 2)$ ways. Thus, $F(n) = F(n - 1) + F(n - 2)$, i.e. this is a sequence of Fibonacci numbers. It is easy to see that $F(1) = 1$ and $F(2) = 2$. Therefore, $F(10) = 89$, the 10th Fibonacci number.

17. Allan and Bill are walking in the same direction beside a railroad track, and Allan is far behind Bill. Both walk at constant speeds, and Allan walks faster than Bill. A long train traveling at a constant speed in the same direction will take 10 seconds to pass Allan (from the front to the end) and will take 9 seconds to pass Bill. If it will take twenty minutes for the front of the train to travel from Allan to Bill, how many minutes will it take for Allan to catch up to Bill?

- A. 200 B. 220 C. 240 D. 260 E. 280

Answer: (a)

Solution: Denote by v_A , v_B , and v_T the speeds of Allan, Bill, and the train, respectively. The relative speeds of the train with respect to Allan and Bill are then $v_T - v_A$ and $v_T - v_B$. It follows from the condition that the train passes Allan in 10 seconds and Bill in 9 seconds that $v_T - v_A / v_T - v_B = 9/10$. The relative speed of Allan with respect to Bill is $v_A - v_B$. We have

$$\frac{v_A - v_B}{v_T - v_B} = \frac{v_T - v_B}{v_T - v_B} - \frac{v_T - v_A}{v_T - v_B} = 1 - \frac{9}{10} = \frac{1}{10}.$$

Thus, Allan’s speed with respect to Bill is ten times smaller than the train’s, so, in order to reach Bill, it would take Allan ten times the time it would take the train, i.e. 200 minutes.

18. In a group of five friends, the sums of the ages of each group of four of them are 124, 128,130, 136, and 142. What is the age of the youngest of the friends?
A. 18 B. 21 C. 23 D. 25 E. 34

Answer: (c)

19. An odd integer between 600 and 800 is divisible by 7 and also divisible by 9. What is the sum of its digits?

- A. 7 B. 12 C. 18 D. 21 E. 27

Answer: (c)

Solution: The integer is a multiple of 63. There are three such multiples between 600 and 800. They are 630, 693, and 756. Only 693 is odd.

20. At night a man who is 6 feet tall stands 5 feet away from a lamppost. The lamppost’s light bulb is 16 feet above the ground. How long is the man’s shadow?
A. 15/8 feet B. 3 feet C. 10/3 feet D. 4 feet E. 5 feet

(b) Let x be the length of the man’s shadow. Considering similar triangles we get $x / x + 5 = 6/16$

21. What is the measure of the acute angle between the hour and minute hands of a correctly working clock at 4:18?
A. 12° B. 15° C. 18° D. 21° E. 24°

Answer: (d)

Solution: Consider an axis through the center of the clock with direction, the direction of the hands of the clock at 12:00. At 4:18 the angle between the axis and the hour hand is $(4 + 18/60) (360/12 = 129^{\circ}$. The angle between the axis and the minute hand is $18/60 (360^{\circ}) = 108^{\circ}$.

22. Jack and Jill collect ladybugs. Jack only collects the ones with 2 spots, and Jill only collects the ones with 7 spots. Jack has 5 more ladybugs than Jill. The total number of spots found on all of their ladybugs is 100. How many ladybugs do they have in their combined collection?

A. 17 B. 21 C. 23 D. 25 E. 35

Answer: (d)

Solution: If x is the number of ladybugs that Jack has and y is the number of ladybugs that Jill has, then $x - y = 5$ and $2x + 7y = 100$. Multiplying the first of these equations by 5 and the second by 2 and adding gives $9(x + y) = 225$. Hence, $x + y = 25$.

23. Jerry had an average score of 85 on his first eight quizzes. He had an average score of 81 on his first nine quizzes. What score did he receive on his ninth quiz?

A. 49 B. 51 C. 53 D. 55 E. 57

Answer: (a) The sum of Jerry's first 8 scores is $8 \cdot 85$ and the sum of his first 9 scores is $9 \cdot 81$, so he received $9 \cdot 81 - 8 \cdot 85$ on his 9th quiz. You can do the arithmetic or note that the answer ends in a 9.

24. The king took a cup filled with water and drank $\frac{1}{5}$ of its contents. When the king looked away, the court jester refilled the cup by adding alcohol to the remaining water and then stirred. The king drank $\frac{1}{4}$ of this liquid mixture. When the king looked away again, the court jester refilled the cup with more alcohol and stirred. The king drank $\frac{1}{3}$ of this liquid mixture. When the king looked away a third time, the court jester refilled the cup with more alcohol. What percent of this final mixture was alcohol?

A. 50% B. 60% C. 70% D. 75% E. 80%

Answer: (b)


Solution: If A is the amount of alcohol present in the mixture at some time and the king drinks a proportion p of the mixture, then the king is drinking a proportion p of each of the mixture's parts and, in particular, of the alcohol. Hence, the amount of alcohol that the king leaves in the cup is $A - Ap = A(1 - p)$. It follows that the proportion of alcohol left in the cup at the end of the problem is

$$\left(\frac{1}{5}\left(1 - \frac{1}{4}\right) + \frac{1}{4}\right)\left(1 - \frac{1}{3}\right) + \frac{1}{3} = \frac{3}{5} = 0.60.$$

25. The length of the shorter side of a rectangle is 2 units. The length of each diagonal is 4 units. What is the acute angle between the diagonals?

A. 15° B. 22.5° C. 45° D. 60° E. 75°

Answer: (d)

Solution: Suppose the vertices of the rectangle are labeled A, B, C, and D in such a way that AB is the shorter side. Let O be the intersection point of the two diagonals. Then $\triangle AOB$ is  equilateral.

26. In a class of 100 students, there are 50 who play soccer, 45 who play basketball, and 50 who play volleyball. Only 15 of these students play all three sports. Everyone plays at least one of these sports. How many of the students play exactly two of these sports?

A. 15 B. 20 C. 25 D. 30 E. 35

Answer: (a)

Solution: Let S_1 be the number of students who play exactly one of the three sports; S_2 – the number of students who play exactly two of the three sports; and let S_3 be the number of students who play all three sports. We know that $S_3 = 15$ and $S_1 + S_2 + S_3 = 100$ (the total number of students). Also in the sum $50+45+50$ (soccer players + basketball players + volleyball players), each student who plays exactly one sport is counted once, the ones who play exactly two sports are counted twice, and those who play all three sports are counted three times. Thus, $50 + 45 + 50 = S_1 + 2S_2 + 3S_3$. Note that $S_2 = (S_1 + 2S_2 + 3S_3) - (S_1 + S_2 + S_3) - 2S_3 = 15$.

27. Fresh grapes contain 80% water by weight, whereas dried grapes contain 15% water by weight. How many pounds of dried grapes can be obtained from 34 pounds of fresh grapes?

- A. 8 B. 9 C. 10 D. E. 12

Answer: (a)

Solution: Suppose we call what's left from a grape after removing all water "waterless content". Then 34 pounds of fresh grapes contain 6.8 pounds "waterless content". Also, x pounds dried grapes contain $x - 0.15x = 0.85x$ pounds "waterless" content". Solving the equation $0.85x = 6.8$ we get $x = 8$.

28. The sum of seven consecutive integers is 980. How many of them are prime?

- A. 0 B. 1 C. 2 D. 3 E. 4

Answer: (c)

Solution: The middle number is 140 and the seven numbers are 137, 138, 139, . . . , 143. Now 138, 140, and 142 are even, 141 is divisible by 3, and 143 - by 11. The remaining numbers 137 and 139 are prime (it is easy to check that they are not divisible by 2,3,5,7 and 11).

29. It rained on exactly 7 of the days during Jane's summer holiday trip. On each day that it rained, it rained either in the morning or the afternoon but not both. There were exactly 5 afternoons when it did not rain and exactly 6 mornings when it did not rain. How many days did the trip last?

- A. 7 B. 8 C. 9 D. 10 E. 11

Answer: (c)

Solution: Let M be the number of days it rained in the morning; A - the number of days it rained in the afternoon; and let N be the number of days when it did not rain. We have $M + A = 7$ (days when it rained); $M + N = 5$ (afternoons when it did not rain); and $A + N = 6$ (mornings when it did not rain). Adding the equations we get $2(M + A + N) = 18$, $M + A + N = 9$.

30. Suppose that 25% of all the wise people are nice and half of all the nice people are wise. Suppose further that 25% of all the people are neither wise nor nice. What percent of all the people are both wise and nice?

- A. 10% B. 15% C. 20% D. 25% E. 30%

Answer: (b)

Solution: Let k be the number of all the people who are both nice and wise. Then, the number of wise people is $4k$ and the number of nice people is $2k$. The number of people who are either wise or nice or both is $4k + 2k - k = 5k$. If the number of all people is x , then $x - 0.25x = 5k$, so $x = 20k/3$.

31. A man caught some fish. The 2 heaviest fish had a combined weight which was 25% of the total weight of all the fish. The 5 lightest fish had a combined weight which was 45% of the total weight of all the fish. He put the 2 heaviest fish in the freezer and ate the 5 lightest fish for lunch. His cat took all the remaining fish. How many fish did his cat take?

- A. 8 B. 6 C. 4 D. 3 E. 2

Answer (d)

Solution: Let W be the total weight of all fish the man caught. The fish taken by the cat had weight $30\%W$. If the cat took 4 fish or more, then their average weight must have been at most $7.5\%W$ which is less than the average weight of the 5 lightest fish. If the cat took 1 or 2 fish, then their average weight must have been at least $15\%W$ which is more than the average weight of the 2 heaviest fish. Thus, the cat took 3 fish.

32. Suppose that 10 teams participated in a soccer tournament where each team played exactly one game with each of the other teams. The winner of each game received 3 points, while the loser received 0 points. In case of a tie, both teams received 1 point. At the end of the tournament, the 10 teams received a total of 130 points. How many games ended in a tie?

- A. 1 B. 2 C. 3 D. 4 E. 5

Answer: (e)

Solution: There were 45 games played in the tournament ($\frac{2}{10} \times 10 \times 10$ games). Now, the total number of points awarded in one game is either 3 (if one team wins), or 2 (when the teams tie the game). So, the total number of points received is $3 \times 45 -$ (the number of games ended in a tie). Thus, exactly 5 games ended in a tie.

33. Jerry wrote down all the positive integers that have at most 7 digits and contain only the digits 0 and 1. How many times did he write down the digit 1?

- A. 128 B. 224 C. 288 D. 448 E. 512

Answer: (d)

Solution: Jerry wrote down a total of $2^7 - 1 = 127$ numbers (0 is missing). Add 0 to the numbers Jerry wrote. We divide the resulting 128 numbers in pairs. If x is one of the numbers Jerry wrote then we put it in pair with $1111111 - x$ (whose digits are also 0s and 1s). For example 1100101 is paired with 11010. In this way, we divide the numbers into 64 distinct pairs so that each number appears exactly once. Note that there are exactly 7 digits 1 in each pair. So, Jerry wrote the digit 1 exactly $64 \times 7 = 448$ times.

34. A group containing boys and girls took a test. If exactly $\frac{2}{3}$ of the boys and exactly $\frac{3}{4}$ of the girls passed the test, and if an equal number of boys and girls passed the test, then what fraction of the entire group passed the test?

- A. $\frac{11}{16}$ B. $\frac{12}{17}$ C. $\frac{13}{18}$ D. $\frac{14}{19}$ E. $\frac{17}{23}$

Answer: (b)

Solution: Let b be the number of boys that took the test and g the number of girls.

We are given that $2b/3 = 3g/4$, so

$b = 9g/8$. So the desired ratio is

$$\frac{(2b/3) + (3g/4)}{b + g} = \frac{2 \cdot (3g/4)}{17g/8} = \frac{3/2}{17/8} = \frac{12}{17}$$

35. How many 5-digit numbers with all digits non-zero and no digit repeated are divisible by 25 ?

- A. 360 B. 420 C. 450 D. 480 E. 500

Answer: (b)

Solution: The 5-digit number must end with 25 or 75 (on the right). The other digits can be arbitrary but must be distinct from each other, the two right-most digits and zero. This leaves $7 \cdot 6 \cdot 5 = 210$ choices for the remaining digits. The answer is $2 \cdot 210 = 420$.

36. Each car of a five-car train must be painted a solid color. The only color choices are red, blue, and yellow. If each of these colors must be used for at least one car, then how many ways are there to paint this train?

- A. 100 B. 125 C. 150 D. 175 E. 200

Answer: (c)

Solution: One can paint five cars using 3 colors in $3^5 = 243$ ways. Also, one can paint five cars using only 2 out of 3 colors in $3 \cdot 2^5 = 96$ ways (there are 3 ways to pick 2 out of 3 colors). Finally, one can paint five cars using only 1 out of 3 colors in $3 \cdot 1^5 = 3$ ways. By the inclusion-exclusion principle, the answer is $243 - 96 + 3 = 150$.

37. What is the number of distinct real numbers x which have the property that the median of the five numbers x ; 6; 4; 1; 9 is equal to their mean?

- A. 0 B. 1 C. 2 D. 3 E. 5

Answer: (d)

Solution: The median cannot be 1 (there are at least three numbers in the set larger than 1). For similar reasons, the median is not 9. The median could be 6. Then $6 = (x+6+4+1+9)/5$, so $x = 10$. Since 6 is the median of the set $\{1; 4; 6; 9; 10\}$ and the mean of the set is also 6, the number 6 has the described property. Further, if the median is 4, then $4 = (x+6+4+1+9)/5$, so $x = 0$. Again, one checks that $x = 0$ has the desired property. Finally, if the median is x , then $x = (x+6+4+1+9)/5$, so $x = 5$, and one checks that $x = 5$ has the desired property, too.

38. The lengths of two sides of a triangle are 2 and 9. Which of the following could be the length of the third side?

- A. 4 B. 6 C. 8 D. 12 E. 14

Answer: (c)

Solution: Three positive numbers are the lengths of the sides of a triangle if and only if each sum of two of the numbers is greater than the third. The choice 8 is the only one of the choices that satisfies this condition

39. If $2^4 \times 3^8 = n \times 6^4$, then $n =$

- A. 12 B. 24 C. 27 D. 54 E. 81

Answer: (e)

Solution: Observing that $6^4 = 2^4 \times 3^4$, one can cancel to obtain $n = 34 = 81$.

40. Which of the following shapes has the largest area?

- (a) A circle with radius of length 3
(b) A square with each side of length 5
(c) A rectangle with sides of lengths 3 and 9
(d) A right triangle with sides of lengths 6, 8, and 10
(e) An equilateral triangle with each side of length 7

Answer: (a)

41. Find three consecutive integers such that the product of the first two is 20 less than the product of the last two.

- A. 7, 8, 9 B. 8,9,10 C. 9, 10, 11 D. 10,11,12 E. 6,7,8

Answer: (c)

Solution: $x(x + 1) = (x + 1)(x + 2) - 20$

$$x^2 + x = x^2 + 3x + 2 - 20$$

$$-2x = -18$$

$$x = 9$$

42. The sum of two numbers is 20. Their difference is 6. What are the numbers?

- A. 7 and 13 B. 8 and 12 C. 5 and 10 D. 6 and 14 E. 9 and 11

Answer: (a)

Solution:

$$x + y = 20$$

$$x - y = 6$$

$$2x = 26$$

$$x = 13$$

$$y = 7$$

43. The grade six pupils at Bayambang Central School went an outing to the National Museum, accompanied by their parents. There were six times as many children as parents. If 245 people went on this outing, how many were parents?

- A. 38 B. 35 C. 42 D. 45 E. 51

Answer: (b)

Solution:

$$P + 6P = 245$$

$$7P = 245$$

P = 35

44. The cost of a full – priced ticket on one airline route is P 3, 750 and the cost of a discounted ticket is P 2, 250. On one flight on this route there were 167 passengers, producing a total ticket income P 557, 250. How many discounted tickets were sold?

- A. 38
- B. 27
- C. 46
- D. 52
- E. 29
- Answer: (c)

Solution: Let x the number of persons with full – priced tickets and let y the number of persons with discounted tickets.

(1) $x + y = 167$

(2) $3\,750\,x + 2\,250\,y = 557\,250$

From equation 1, we have $x = 167 - y$

By substitution, we have $3\,750\,(167 - y) + 2\,250\,y = 557\,250$

$y = 46$

45. $\frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \frac{1}{12},$ _____

- A. $\frac{1}{14}$
- B. $\frac{1}{18}$
- C. $\frac{1}{20}$
- D. $\frac{1}{24}$
- E. $\frac{1}{28}$
- Answer: (d)

Solution: Succeeding, $\times 2$

46. $\frac{3}{5} + \frac{1}{4} + \frac{2}{3} = ?$

- A. $1\frac{33}{60}$
- B. $2\frac{29}{60}$
- C. $1\frac{13}{60}$
- D. $2\frac{17}{60}$
- E. $1\frac{11}{60}$
- Answer: (a)

47. Which of the following has least numerical value?

- A. $\frac{1}{3}$
- B. $\frac{2}{7}$
- C. $\frac{3}{5}$
- D. $\frac{4}{9}$
- E. $\frac{5}{12}$
- Answer: (b)

48. $4 + 2 (3 + 6) \div 3 - 1 = ?$

- A. 6
- B. 7
- C. 8
- D. 9
- E. 10
- Answer: (9)

49. $\frac{1}{3} + \frac{2}{7} + \frac{1}{2} = ?$

- A. $1\frac{5}{42}$
- B. $2\frac{3}{22}$
- C. $1\frac{7}{27}$
- D. $2\frac{3}{19}$
- E. $1\frac{4}{11}$
- Answer: (a)

50. By how much larger is 3 than – 7?

- A. – 10
- B. 10
- C. 4
- D. – 4
- E. 0
- Answer: (b)

_____1. $16 + 4 \times (7 + 8) - 3 =$ _____?

- a. 117
- b. 145
- c. 73
- d. 65

_____ 2. $(18 + 17) (12 + 9) - (7 \times 16) (4 + 2) =$ _____?

- a. 53
- b. 63
- c. 321
- d. 323

_____ 3. The sum of 73, 2891, 406 and 98 is _____?

- a. 3468
- b. 3486
- c. 3648
- d. 4648

_____ 4. Which of the following numbers is divisible by 24 ?

- a. 192
- b. 268
- c. 248
- d. 596

_____ 5. Which of the following numbers is prime?

- a. 57
- b. 87
- c. 89
- d. 91

Solutions

1. $16 + 4 \times (7 + 8) - 3 =$ _____?

$$= 16 + 4 \times (15) - 3$$

$$= 16 + 60 - 3$$

$$= 16 + 57$$

= 73 *Ans.

2. $(18 + 17) (12 + 9) - (7 \times 16) (4 + 2) =$ _____?

$$= (35) (21) - (112) (6)$$

$$= 735 - 672$$

= 63 *Ans.

3. The sum of 73, 2891, 406 and 98 is _____?

$$= 73 + 2891 + 406 + 98$$

= 3468 *Ans.

4. Which of the following numbers is divisible by 24 ?

192 ÷ 24 = 8 *Ans.

286 ÷ 24 = 11 remainder 4, 268 not divisible by 24 because it has a remainder when divided by 24.

5. Which of the following numbers is prime?

a. 57 = 3 x 19

b. 87 = 3 x 29

c. 89 = 89 x 1 *Ans.

d. 91 = 13 x 7

_____ 1. 27, 499 round to the nearest hundred is _____?

- a. 27, 400
- b. 27, 500
- c. 27, 000
- d. 28, 000

_____ 2. Twenty-four weeks is how many days?

- a. 140
- b. 168
- c. 176
- d. 196

_____ 3. Five hundred ninety-five days is how many weeks?

- a. 119
- b. 95
- c. 85
- d. 75

_____ 4. Eighteen bus loads of 56 students each went to join the Independence Day Celebration. One hundred seventy-four did not go. How many students are there in all?

- a. 160
- b. 1282
- c. 180
- d. 1182

_____ 5. Richard bowled 3 games and got scores of 139, 153, and 128. What was his average score for the three games?

- a. 130
- b. 140
- c. 150

d. 160

1. 27, 499 round to the nearest hundred is _____?

= 27, 499 ---> drop 99 and change it to 00 and add 1 to the next digit which is 4 since 99 is more than 50.

Therefore **27,500 is the answer. *Ans.**

2. Twenty-four weeks is how many days?

$$= 24 \text{ weeks} \times \frac{7 \text{ days}}{\text{week}}$$

$$= 24 \times 7 \text{ days}$$

= 168 days *Ans.

3. Five hundred ninety-five days is how many weeks?

$$= 595 \text{ days} \times \frac{1 \text{ week}}{7 \text{ days}}$$

$$= \frac{595}{7} \text{ week}$$

= 85 weeks *Ans.

4. Eighteen bus loads of 56 students each went to join the Independence Day Celebration. One hundred seventy-four did not go. How many students are there in all?

Number of students :

$$N = (18 \times 56) + 174$$

$$= 1008 + 174$$

= 1182 students *Ans.

5. Richard bowled 3 games and got scores of 139, 153, and 128. What was his average score for the three games?

$$\text{Average} = \frac{139 + 153 + 128}{3}$$

$$= \frac{420}{3}$$

= 140 *Ans.

_____ 1. If $9x - 7 = 18y$ then $\underline{9x - 7} = \underline{\hspace{2cm}}$?

- a. $2y$
- b. $3y$
- c. $6y$
- d. $y + 6$

_____ 2. A student buys an MSA Reviewer Book for \$175 after receiving a discount of 12.5%. What was the marked price?

- a. \$ 187.50
- b. \$ 200
- c. \$ 225
- d. \$ 250

_____ 3. A town house unit was sold for \$2.50 M, yielding a 25% profit. For how much would it be sold to yield only a 10% profit on the cost?

- a. \$ 2M
- b. \$ 2.25M
- c. \$ 2.2M
- d. \$2.45M

_____ 4. What single discount is equivalent to successive discounts of 5% and 10%?

- a. 10.5%
- b. 12.5%
- c. 14.5%
- d. 15%

_____ 5. How many miles are there in 40 kilometers?

- a. 25
- b. 64
- c. 32
- d. 60

Solutions

1. If $9x - 7 = 18y$ then $\frac{9x - 7}{6} = \underline{\hspace{2cm}}?$

$$9x - 7 = 18y$$

$$\frac{9x - 7}{6} = \frac{18y}{6}$$

= 3y * Ans.

2. A student buys an MSA Reviewer Book for \$175 after receiving a discount of 12.5%. What was the marked price?

$$1 \text{ MP} - 0.125 \text{ MP} = \$175$$

$$0.875 \text{ MP} = \$175$$

$$7/8 \text{ MP} = \$175 \text{ (note : } 7/8 \text{ is the fraction form of } 0.875)$$

$$\text{MP} = \$175 \times (8/7)$$

MP = \$200 *Ans.

3. A town house unit was sold fir \$2.50 M, yielding a 25% profit. For how much would it be sold to yield only a 10% profit on the cost?

Let C be the original cost of the house
Let 0.25C be the profit

$C + 0.25C = \$2.5\text{ M}$

$1.25C = \$2.5\text{M}$

$\frac{1.25C}{1.25} = \frac{\$2.5\text{M}}{1.25}$

$C = \$2\text{M}$

The new selling price that would yield at 10% profit on the cost would be :

$= \$2\text{M} + 0.10 (\$2\text{M})$
 $= \$2\text{M} + 0.20\text{M}$
= \$2.2M *Ans.

4. What single discount is equivalent to successive discounts if 5% and 10%?

The formula for a single rate of discount equivalent to the series of discounts is :

$R = 1 - [(1 - r_1) (1 - r_2) \dots (1 - r_n)]$

for $r_1 = 5\%$ and $r_2 = 10\%$

$R = 1 - [(1 - 0.05) (1 - 0.10)]$
 $= 1 - [(0.95) (0.90)]$
 $= 1 - 0.855$
= 0.145 or 14.5% *Ans.

5. How many miles are there in 40 kilometers?

$1\text{ mile} = 1.6\text{ kilometers}$

$40\text{ kilometers} = 40\text{ km} \times \frac{1\text{ mile}}{1.6\text{ km}}$

= 25 miles *Ans.

Select the best answer for each and rite the appropriate letter in the blank.

_____1. A car that cost \$ 1.2 M can be sold for \$ 600, 000 after 5 years of use. What will be the yearly depreciation cost ?

a. \$ 100, 000

b. \$ 12, 000

c. \$ 120, 000

d. \$ 600, 000

_____2. How many times does the digit 7 appear in the numbers from 1 to 100 ?

a. 9

- b. 10
- c. 19
- d. 20

3. At the rate of \$ 44 per hundred sheets of colored bond paper, how much is the cost of 500 sheets ?

- a. \$ 121
- b. \$ 242
- c. \$ 440
- d. \$ 480

4. At \$ 25 per board foot of wood, what is the cost of 15 pieces of 2" x 2" x 12' ?
(1 board foot = 1 ft. x 1 ft. x 1 inch)

- a. \$ 18, 000
- b. \$ 15, 000
- c. \$ 1, 250
- d. \$ 1, 500

5. The decimal form of 0.56 % is _____ ?

- a. 0.0056
- b. 0.056
- c. 0.56
- d. 56

1. A car that cost \$ 1.2 M can be sold for \$ 600, 000 after 5 years of use. What will be the yearly depreciation cost ?

Yearly depreciation cost = $\frac{\text{depreciation}}{\text{no. of years}}$

$$= \frac{\$ 1,200,000 - \$ 600,000}{5}$$

$$= \frac{\$ 600,000}{5}$$

= \$ 120, 000 per year * Ans.

2. How many times does the digit 7 appear in the numbers from 1 to 100 ?

7, 17, 27, 37, 47, 57, 67, 77, 87, 97,

71, 72, 73, 74, 75, 76, 77, 78, 79

19 times * Ans.

3. At the rate of \$ 44 per hundred sheets of colored bond paper, how much is the cost of 500 sheets ?

Cost of the 550 sheets = $\frac{\$ 44}{100 \text{ sheets}} \times 550 \text{ sheets}$

$$= \$ 44 \times 5.5$$

= \$ 242 * Ans.

4. At \$ 25 per board foot of wood, what is the cost of 15 pieces of 2" x 2" x 12' ?

(1 board foot = 1 ft. x 1 ft. x 1 inch)

$$\text{Cost} = \frac{\$ 25}{\text{bd. ft.}} \times \frac{2" \times 2" \times 12"}{12" \text{ bd. ft.}}$$

$$= \$ 1, 500 \text{ * Ans.}$$

5. The decimal form of 0.56 % is _____ ?

Move 2 decimal places to the left (100%).

$$0.56 \% = 0.0056 \text{ * Ans.}$$

_____ 1. What number is as much more than 8 as it is less than 32 ?

- a. 20

b. 40

c. 60

d. cannot be determined from the given information
- _____ 2. A container van that is 3 meters wide, 5 meters long and 4 meters high will transport 200 crates whose volume is 6 cubic meters. How many trips will it take to transport all the crates?
- a. 20

b. 25

c. 30

d. 35
- _____ 3. A rectangular block of copper, with dimensions 4m x 6m x 9m, is melted and recast into a cubical block. Find the length of the side of the cubical block.
- a. 4 cm

b. 6 cm

c. 9 cm

d. 12 cm
- _____ 4. There are 9 male teachers for every 14 female teachers. If there are 69 teachers in all, how many teachers are female?
- a. 18

b. 27

c. 39

d. 42
- _____ 5. What would be the closest approximation to $\sqrt{66}$?

- a. 7.9

b. 8

c. 8.1

d. 8.9

1.less than 32 ?

Let x - be the number

$$\begin{aligned}
 x - 8 &= 32 - x \\
 x + x &= 32 + 8 \\
 2x &= 40 \\
 x &= 40 / 2 \\
 \mathbf{x = 20 * Ans.}
 \end{aligned}$$

- e. 2. A container van that is 3 meters wide, 5 meters long and 4 meters high will transport 200 crates whose volume is 6 cubic meters. How many trips will it take to transport all the crates?

Let N - be the number of trips

$$\text{No. of trips} = (\text{No. of crates}) \times \frac{\text{volume}}{\text{crate}} \times \frac{1 \text{ trip}}{\text{volume of the van}}$$

$$N = 200 \text{ crates} \times \frac{6 \text{ m}^3}{\text{crate}} \times \frac{1 \text{ trip}}{(3 \times 5 \times 4)\text{m}^3}$$

$$N = \frac{(200) (6)}{3 \times 4 \times 5}$$

$$\begin{aligned}
 N &= \frac{1200}{60} \\
 \mathbf{N = 20 trips * Ans.}
 \end{aligned}$$

- f. _____
g. 3. A rectangular block of copper, with dimensions 4m x 6m x 9m, is melted and recast into a cubical block. Find the length of the side of the cubical block.

$$V_{\text{cubical block}} = V_{\text{rectangular block}}$$

$$\begin{aligned}
 S^3 &= (4 \times 6 \times 9) \text{ m}^3 \\
 S^3 &= 216 \text{ m}^3
 \end{aligned}$$

$$S = \sqrt[3]{216 \text{ m}^3}$$

$$\mathbf{S = 6m * Ans.}$$

- h. _____
i. 4. There are 9 male teachers for every 14 female teachers. If there are 69 teachers in all, how many teachers are female?

$$\frac{9 : 14}{23} = \frac{\text{Male : Female}}{69}$$

$$\begin{aligned}
 \text{No. of Female teachers} &= (69 / 23) \times 14 \\
 &= 3 \times 14 \\
 \mathbf{= 42 * Ans.}
 \end{aligned}$$

- j. _____
k. 5. What would be the closest approximation to $\sqrt{66}$?

$$\sqrt{64} = 8$$

$$\mathbf{\sqrt{66} \cong 8.1 * Ans.}$$

- l. _____

